Question ID 91e7ea5e

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	■■■

ID: 91e7ea5e

$h(x) = 2(x-4)^2 - 32$

The quadratic function h is defined as shown. In the xy-plane, the graph of $y = h(x)$ intersects the x-axis at the points $(0,0)$ and $(t,0)$, where t is a constant. What is the value of t ?

A. 1

B. 2

C. 4

D. 8

ID: 91e7ea5e Answer

Correct Answer: D

Rationale

Choice D is correct. It's given that the graph of $y = h(x)$ intersects the x-axis at $(0,0)$ and $(t,0)$, where t is a constant. Since this graph intersects the x-axis when $y = 0$ or when $h(x) = 0$, it follows that $h(0) = 0$ and $h(t) = 0$. If $h(t) = 0$, then $0 = 2(t-4)^2 - 32$. Adding 32 to both sides of this equation yields $32 = 2(t-4)^2$. Dividing both sides of this equation by 2 yields $16 = (t-4)^2$. Taking the square root of both sides of this equation yields $4 = t - 4$. Adding 4 to both sides of this equation yields $8 = t$. Therefore, the value of t is 8.

Choices A, B, and C are incorrect and may result from calculation errors.

Question Difficulty: Hard

Question ID a9084ca4

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	■■■

ID: a9084ca4

$$f(x) = 9{,}000(0.66)^x$$

The given function f models the number of advertisements a company sent to its clients each year, where x represents the number of years since 1997, and $0 \le x \le 5$. If $y = f(x)$ is graphed in the *xy*-plane, which of the following is the best interpretation of the *y*-intercept of the graph in this context?

A. The minimum estimated number of advertisements the company sent to its clients during the 5 years was $1{,}708$.

B. The minimum estimated number of advertisements the company sent to its clients during the 5 years was $9{,}000$.

C. The estimated number of advertisements the company sent to its clients in 1997 was $1{,}708$.

D. The estimated number of advertisements the company sent to its clients in 1997 was $9{,}000$.

ID: a9084ca4 Answer

Correct Answer: D

Rationale

Choice D is correct. The *y*-intercept of a graph in the *xy*-plane is the point where $x = 0$. For the given function f, the *y*-intercept of the graph of $y = f(x)$ in the *xy*-plane can be found by substituting 0 for x in the equation $y = 9{,}000(0.66)^x$, which gives $y = 9{,}000(0.66)^0$. This is equivalent to $y = 9{,}000(1)$, or $y = 9{,}000$. Therefore, the *y*-intercept of the graph of $y = f(x)$ is $(0, 9{,}000)$. It's given that the function f models the number of advertisements a company sent to its clients each year. Therefore, $f(x)$ represents the estimated number of advertisements the company sent to its clients each year. It's also given that x represents the number of years since 1997. Therefore, $x = 0$ represents 0 years since 1997, or 1997. Thus, the best interpretation of the *y*-intercept of the graph of $y = f(x)$ is that the estimated number of advertisements the company sent to its clients in 1997 was $9{,}000$.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID b8f13a3a

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	■■■

ID: b8f13a3a

Function f is defined by $f(x) = -a^x + b$, where a and b are constants. In the *xy*-plane, the graph of $y = f(x) - 12$ has a *y*-intercept at $\left(0, -\frac{75}{7}\right)$. The product of a and b is $\frac{320}{7}$. What is the value of a?

ID: b8f13a3a Answer

Correct Answer: 20

Rationale

The correct answer is 20. It's given that $f(x) = -a^x + b$. Substituting $-a^x + b$ for $f(x)$ in the equation $y = f(x) - 12$ yields $y = -a^x + b - 12$. It's given that the y-intercept of the graph of $y = f(x) - 12$ is $\left(0, -\frac{75}{7}\right)$. Substituting 0 for x and $-\frac{75}{7}$ for y in the equation $y = -a^x + b - 12$ yields $-\frac{75}{7} = -a^0 + b - 12$, which is equivalent to $-\frac{75}{7} = -1 + b - 12$, or $-\frac{75}{7} = b - 13$. Adding 13 to both sides of this equation yields $\frac{16}{7} = b$. It's given that the product of a and b is $\frac{320}{7}$, or $ab = \frac{320}{7}$. Substituting $\frac{16}{7}$ for b in this equation yields $(a)\left(\frac{16}{7}\right) = \frac{320}{7}$. Dividing both sides of this equation by $\frac{16}{7}$ yields $a = 20$.

Question Difficulty: Hard

Question ID 7902bed0

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	■■■

ID: 7902bed0

A machine launches a softball from ground level. The softball reaches a maximum height of 51.84 meters above the ground at 1.8 seconds and hits the ground at 3.6 seconds. Which equation represents the height above ground h, in meters, of the softball t seconds after it is launched?

A. $h = -t^2 + 3.6$

B. $h = -t^2 + 51.84$

C. $h = -16(t + 51.84)^2 - 3.6$

D. $h = -16(t - 1.8)^2 + 51.84$

ID: 7902bed0 Answer

Correct Answer: D

Rationale

Choice D is correct. An equation representing the height above ground h, in meters, of a softball t seconds after it is launched by a machine from ground level can be written in the form $h = -a(t - b)^2 + c$, where a, b, and c are positive constants. In this equation, b represents the time, in seconds, at which the softball reaches its maximum height of c meters above the ground. It's given that this softball reaches a maximum height of 51.84 meters above the ground at 1.8 seconds; therefore, $b = 1.8$ and $c = 51.84$. Substituting 1.8 for b and 51.84 for c in the equation $h = -a(t - b)^2 + c$ yields $h = -a(t - 1.8)^2 + 51.84$. It's also given that this softball hits the ground at 3.6 seconds; therefore, $h = 0$ when $t = 3.6$. Substituting 0 for h and 3.6 for t in the equation $h = -a(t - 1.8)^2 + 51.84$ yields $0 = -a(3.6 - 1.8)^2 + 51.84$, which is equivalent to $0 = -a(1.8)^2 + 51.84$, or $0 = -3.24a + 51.84$. Adding $3.24a$ to both sides of this equation yields $3.24a = 51.84$. Dividing both sides of this equation by 3.24 yields $a = 16$. Substituting 16 for a in the equation $h = -a(t - 1.8)^2 + 51.84$ yields $h = -16(t - 1.8)^2 + 51.84$. Therefore, $h = -16(t - 1.8)^2 + 51.84$ represents the height above ground h, in meters, of this softball t seconds after it is launched.

Choice A is incorrect. This equation represents a situation where the maximum height is 3.6 meters above the ground at 0 seconds, not 51.84 meters above the ground at 1.8 seconds.

Choice B is incorrect. This equation represents a situation where the maximum height is 51.84 meters above the ground at 0 seconds, not 1.8 seconds.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 4a0d0399

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	■■■

ID: 4a0d0399

The function f is defined by $f(x) = a^x + b$, where a and b are constants. In the xy-plane, the graph of $y = f(x)$ has an x-intercept at $(2, 0)$ and a y-intercept at $(0, -323)$. What is the value of b?

ID: 4a0d0399 Answer

Correct Answer: -324

Rationale

The correct answer is -324. It's given that the function f is defined by $f(x) = a^x + b$, where a and b are constants. It's also given that the graph of $y = f(x)$ has a y-intercept at $(0, -323)$. It follows that $f(0) = -323$. Substituting 0 for x and -323 for $f(x)$ in $f(x) = a^x + b$ yields $-323 = a^0 + b$, or $-323 = 1 + b$. Subtracting 1 from each side of this equation yields $-324 = b$. Therefore, the value of b is -324.

Question Difficulty: Hard

Question ID 9654add7

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	■■■

ID: 9654add7

$f(x) = -500x^2 + 25{,}000x$

The revenue $f(x)$, in dollars, that a company receives from sales of a product is given by the function *f* above, where *x* is the unit price, in dollars, of the product. The graph of $y = f(x)$ in the *xy*-plane intersects the *x*-axis at 0 and *a*. What does *a* represent?

A. The revenue, in dollars, when the unit price of the product is $0

B. The unit price, in dollars, of the product that will result in maximum revenue

C. The unit price, in dollars, of the product that will result in a revenue of $0

D. The maximum revenue, in dollars, that the company can make

ID: 9654add7 Answer

Correct Answer: C

Rationale

Choice C is correct. By definition, the y-value when a function intersects the x-axis is 0. It's given that the graph of the function intersects the x-axis at 0 and a, that x is the unit price, in dollars, of a product, and that $f(x)$, where $y = f(x)$, is the revenue, in dollars, that a company receives from the sales of the product. Since the value of a occurs when $y = 0$, a is the unit price, in dollars, of the product that will result in a revenue of $0.

Choice A is incorrect. The revenue, in dollars, when the unit price of the product is $0 is represented by $f(x)$, when $x = 0$. Choice B is incorrect. The unit price, in dollars, of the product that will result in maximum revenue is represented by the x-coordinate of the maximum of f. Choice D is incorrect. The maximum revenue, in dollars, that the company can make is represented by the y-coordinate of the maximum of f.

Question Difficulty: Hard

Question ID 263f9937

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	■■■

ID: 263f9937

Growth of a Culture of Bacteria

Day	Number of bacteria per milliliter at end of day
1	2.5×10^5
2	5.0×10^5
3	1.0×10^6

A culture of bacteria is growing at an exponential rate, as shown in the table above. At this rate, on which day would the number of bacteria per milliliter reach 5.12×10^8?

A. Day 5

B. Day 9

C. Day 11

D. Day 12

ID: 263f9937 Answer

Correct Answer: D

Rationale

Choice D is correct. The number of bacteria per milliliter is doubling each day. For example, from day 1 to day 2, the number of bacteria increased from 2.5×10^5 to 5.0×10^5. At the end of day 3 there are 10^6 bacteria per milliliter. At the end of day 4, there will be $10^6 \times 2$ bacteria per milliliter. At the end of day 5, there will be $(10^6 \times 2) \times 2$, or $10^6 \times (2^2)$ bacteria per milliliter, and so on. At the end of day d, the number of bacteria will be $10^6 \times (2^{d-3})$. If the number of bacteria per milliliter will reach 5.12×10^8 at the end of day d, then the equation $10^6 \times (2^{d-3}) = 5.12 \times 10^8$ must hold. Since 5.12×10^8 can be rewritten as 512×10^6, the equation is equivalent to $2^{d-3} = 512$. Rewriting 512 as 2^9 gives $d - 3 = 9$, so $d = 12$. The number of bacteria per milliliter would reach 5.12×10^8 at the end of day 12.

Choices A, B, and C are incorrect. Given the growth rate of the bacteria, the number of bacteria will not reach 5.12×10^8 per milliliter by the end of any of these days.

Question Difficulty: Hard

Question ID 18e35375

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	■■■

ID: 18e35375

$$f(x) = (x - 14)(x + 19)$$

The function f is defined by the given equation. For what value of x does $f(x)$ reach its minimum?

A. -266

B. -19

C. $-\frac{33}{2}$

D. $-\frac{5}{2}$

ID: 18e35375 Answer

Correct Answer: D

Rationale

Choice D is correct. It's given that $f(x) = (x - 14)(x + 19)$, which can be rewritten as $f(x) = x^2 + 5x - 266$. Since the coefficient of the x^2-term is positive, the graph of $y = f(x)$ in the *xy*-plane opens upward and reaches its minimum value at its vertex. The *x*-coordinate of the vertex is the value of x such that $f(x)$ reaches its minimum. For an equation in the form $f(x) = ax^2 + bx + c$, where a, b, and c are constants, the *x*-coordinate of the vertex is $-\frac{b}{2a}$. For the equation $f(x) = x^2 + 5x - 266$, $a = 1$, $b = 5$, and $c = -266$. It follows that the *x*-coordinate of the vertex is $-\frac{5}{2(1)}$, or $-\frac{5}{2}$. Therefore, $f(x)$ reaches its minimum when the value of x is $-\frac{5}{2}$.

Alternate approach: The value of x for the vertex of a parabola is the *x*-value of the midpoint between the two *x*-intercepts of the parabola. Since it's given that $f(x) = (x - 14)(x + 19)$, it follows that the two *x*-intercepts of the graph of $y = f(x)$ in the *xy*-plane occur when $x = 14$ and $x = -19$, or at the points $(14, 0)$ and $(-19, 0)$. The midpoint between two points, (x_1, y_1) and (x_2, y_2), is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$. Therefore, the midpoint between $(14, 0)$ and $(-19, 0)$ is $\left(\frac{14+(-19)}{2}, \frac{0+0}{2}\right)$, or $\left(-\frac{5}{2}, 0\right)$. It follows that $f(x)$ reaches its minimum when the value of x is $-\frac{5}{2}$.

Choice A is incorrect. This is the *y*-coordinate of the *y*-intercept of the graph of $y = f(x)$ in the *xy*-plane.

Choice B is incorrect. This is one of the *x*-coordinates of the *x*-intercepts of the graph of $y = f(x)$ in the *xy*-plane.

Choice C is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 9afe2370

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	■■■

ID: 9afe2370

The population P of a certain city y years after the last census is modeled by the equation below, where r is a constant and P_0 is the population when $y = 0$.

$P = P_0(1 + r)^y$

If during this time the population of the city decreases by a fixed percent each year, which of the following must be true?

A. $r < -1$

B. $-1 < r < 0$

C. $0 < r < 1$

D. $r > 1$

ID: 9afe2370 Answer

Correct Answer: B

Rationale

Choice B is correct. The term (1 + r) represents a percent change. Since the population is decreasing, the percent change must be between 0% and 100%. When the percent change is expressed as a decimal rather than as a percent, the percentage change must be between 0 and 1. Because (1 + r) represents percent change, this can be expressed as $0 < 1 + r < 1$. Subtracting 1 from all three terms of this compound inequality results in $-1 < r < 0$.

Choice A is incorrect. If $r < -1$, then after 1 year, the population P would be a negative value, which is not possible. Choices C and D are incorrect. For any value of $r > 0$, $1 + r > 1$, and the exponential function models growth for positive values of the exponent. This contradicts the given information that the population is decreasing.

Question Difficulty: Hard

Question ID 0121a235

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	■■■

ID: 0121a235

x	$p(x)$
-2	5
-1	0
0	-3
1	-1
2	0

The table above gives selected values of a polynomial function p. Based on the values in the table, which of the following must be a factor of p ?

A. $(x-3)$

B. $(x+3)$

C. $(x-1)(x+2)$

D. $(x+1)(x-2)$

ID: 0121a235 Answer

Correct Answer: D

Rationale

Choice D is correct. According to the table, when x is -1 or 2, $p(x)=0$. Therefore, two x-intercepts of the graph of p are $(-1,0)$ and $(2,0)$. Since $(-1,0)$ and $(2,0)$ are x-intercepts, it follows that $(x+1)$ and $(x-2)$ are factors of the polynomial equation. This is because when $x=-1$, the value of $x+1$ is 0. Similarly, when $x=2$, the value of $x-2$ is 0. Therefore, the product $(x+1)(x-2)$ is a factor of the polynomial function p.

Choice A is incorrect. The factor $x-3$ corresponds to an x-intercept of $(3,0)$, which isn't present in the table. Choice B is incorrect. The factor $x+3$ corresponds to an x-intercept of $(-3,0)$, which isn't present in the table. Choice C is incorrect. The factors $x-1$ and $x+2$ correspond to x-intercepts $(1,0)$ and $(-2,0)$, respectively, which aren't present in the table.

Question Difficulty: Hard

Question ID 70753f99

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	■■■

ID: 70753f99

The function *f* is defined by $f(x) = (x+3)(x+1)$. The graph of *f* in the *xy*-plane is a parabola. Which of the following intervals contains the *x*-coordinate of the vertex of the graph of *f* ?

A. $-4 < x < -3$

B. $-3 < x < 1$

C. $1 < x < 3$

D. $3 < x < 4$

ID: 70753f99 Answer

Correct Answer: B

Rationale

Choice B is correct. The graph of a quadratic function in the xy-plane is a parabola. The axis of symmetry of the parabola passes through the vertex of the parabola. Therefore, the vertex of the parabola and the midpoint of the segment between the two x-intercepts of the graph have the same x-coordinate. Since $f(-3) = f(-1) = 0$, the x-coordinate of the vertex is $\frac{(-3)+(-1)}{2} = -2$. Of the shown intervals, only the interval in choice B contains –2. Choices A, C, and D are incorrect and may result from either calculation errors or misidentification of the graph's x-intercepts.

Question Difficulty: Hard

Question ID 58dcc59f

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	■■■

ID: 58dcc59f

A landscaper is designing a rectangular garden. The length of the garden is to be 5 feet longer than the width. If the area of the garden will be 104 square feet, what will be the length, in feet, of the garden?

ID: 58dcc59f Answer

Rationale

The correct answer is 13. Let w represent the width of the rectangular garden, in feet. Since the length of the garden will be 5 feet longer than the width of the garden, the length of the garden will be $w+5$ feet. Thus the area of the garden will be $w(w+5)$. It is also given that the area of the garden will be 104 square feet. Therefore, $w(w+5)=104$, which is equivalent to $w^2+5w-104=0$. Factoring this equation results in $(w+13)(w-8)=0$. Therefore, $w=8$ and $w=-13$. Because width cannot be negative, the width of the garden must be 8 feet. This means the length of the garden must be $8+5=13$ feet.

Question Difficulty: Hard

Question ID 84dd43f8

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	■■■

ID: 84dd43f8

For the function f, $f(0) = 86$, and for each increase in x by 1, the value of $f(x)$ decreases by 80%. What is the value of $f(2)$?

ID: 84dd43f8 Answer

Correct Answer: 3.44, 86/25

Rationale

The correct answer is 3.44. It's given that $f(0) = 86$ and that for each increase in x by 1, the value of $f(x)$ decreases by 80%. Because the output of the function decreases by a constant percentage for each 1-unit increase in the value of x, this relationship can be represented by an exponential function of the form $f(x) = a(b)^x$, where a represents the initial value of the function and b represents the rate of decay, expressed as a decimal. Because $f(0) = 86$, the value of a must be 86. Because the value of $f(x)$ decreases by 80% for each 1-unit increase in x, the value of b must be $(1 - 0.80)$, or 0.2. Therefore, the function f can be defined by $f(x) = 86(0.2)^x$. Substituting 2 for x in this function yields $f(2) = 86(0.2)^2$, which is equivalent to $f(2) = 86(0.04)$, or $f(2) = 3.44$. Either 3.44 or $86/25$ may be entered as the correct answer.

Alternate approach: It's given that $f(0) = 86$ and that for each increase in x by 1, the value of $f(x)$ decreases by 80%. Therefore, when $x = 1$, the value of $f(x)$ is $(100 - 80)\%$, or 20%, of 86, which can be expressed as $(0.20)(86)$. Since $(0.20)(86) = 17.2$, the value of $f(1)$ is 17.2. Similarly, when $x = 2$, the value of $f(x)$ is 20% of 17.2, which can be expressed as $(0.20)(17.2)$. Since $(0.20)(17.2) = 3.44$, the value of $f(2)$ is 3.44. Either 3.44 or $86/25$ may be entered as the correct answer.

Question Difficulty: Hard

Question ID 59d1f4b5

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	■■■

ID: 59d1f4b5

$M = 1{,}800(1.02)^t$

The equation above models the number of members, *M,* of a gym *t* years after the gym opens. Of the following, which equation models the number of members of the gym *q* quarter years after the gym opens?

A. $M = 1{,}800(1.02)^{\frac{q}{4}}$

B. $M = 1{,}800(1.02)^{4q}$

C. $M = 1{,}800(1.005)^{4q}$

D. $M = 1{,}800(1.082)^{q}$

ID: 59d1f4b5 Answer

Correct Answer: A

Rationale

Choice A is correct. In 1 year, there are 4 quarter years, so the number of quarter years, q, is 4 times the number of years, t ; that is, $q = 4t$. This is equivalent to $t = \frac{q}{4}$, and substituting this into the expression for M in terms of t gives $M = 1{,}800(1.02)^{\frac{q}{4}}$.

Choices B and D are incorrect and may be the result of incorrectly using $t = 4q$ instead of $q = 4t$. (Choices B and D are nearly the same since 1.02^{4q} is equivalent to $(1.02^4)^q$, which is approximately 1.082^q.) Choice C is incorrect and may be the result of incorrectly using $t = 4q$ and unnecessarily dividing 0.02 by 4.

Question Difficulty: Hard

Question ID 01668cd6

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	■■■

ID: 01668cd6

The functions f and g are defined by the given equations, where $x \geq 0$. Which of the following equations displays, as a constant or coefficient, the maximum value of the function it defines, where $x \geq 0$?

I. $f(x) = 33(0.4)^{x+3}$
II. $g(x) = 33(0.16)(0.4)^{x-2}$

A. I only

B. II only

C. I and II

D. Neither I nor II

ID: 01668cd6 Answer

Correct Answer: B

Rationale

Choice B is correct. Functions f and g are both exponential functions with a base of 0.40. Since 0.40 is less than 1, functions f and g are both decreasing exponential functions. This means that $f(x)$ and $g(x)$ decrease as x increases. Since $f(x)$ and $g(x)$ decrease as x increases, the maximum value of each function occurs at the least value of x for which the function is defined. It's given that functions f and g are defined for $x \geq 0$. Therefore, the maximum value of each function occurs at $x = 0$. Substituting 0 for x in the equation defining f yields $f(0) = 33(0.4)^{0+3}$, which is equivalent to $f(0) = 33(0.4)^3$, or $f(0) = 2.112$. Therefore, the maximum value of f is 2.112. Since the equation $f(x) = 33(0.4)^{x+3}$ doesn't display the value 2.112, the equation defining f doesn't display the maximum value of f. Substituting 0 for x in the equation defining g yields $g(0) = 33(0.16)(0.4)^{0-2}$, which can be rewritten as $g(0) = 33(0.16)\left(\frac{1}{0.4^2}\right)$, or $g(0) = 33(0.16)\left(\frac{1}{0.16}\right)$, which is equivalent to $g(0) = 33$. Therefore, the maximum value of g is 33. Since the equation $g(x) = 33(0.16)(0.4)^{x-2}$ displays the value 33, the equation defining g displays the maximum value of g. Thus, only equation II displays, as a constant or coefficient, the maximum value of the function it defines.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice C is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 635f54ee

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	■■■

ID: 635f54ee

The surface area of a cube is $6\left(\frac{a}{4}\right)^2$, where *a* is a positive constant. Which of the following gives the perimeter of one face of the cube?

A. $\frac{a}{4}$

B. a

C. $4a$

D. $6a$

ID: 635f54ee Answer

Correct Answer: B

Rationale

Choice B is correct. A cube has 6 faces of equal area, so if the total surface area of a cube is $6\left(\frac{a}{4}\right)^2$, then the area of one face is $\left(\frac{a}{4}\right)^2$. Likewise, the area of one face of a cube is the square of one of its edges; therefore, if the area of one face is $\left(\frac{a}{4}\right)^2$, then the length of one edge of the cube is $\frac{a}{4}$. Since the perimeter of one face of a cube is four times the length of one edge, the perimeter is $4\left(\frac{a}{4}\right) = a$.

Choice A is incorrect because if the perimeter of one face of the cube is $\frac{a}{4}$, then the total surface area of the cube is $6\left(\frac{\frac{a}{4}}{4}\right)^2 = 6\left(\frac{a}{16}\right)^2$, which is not $6\left(\frac{a}{4}\right)^2$. Choice C is incorrect because if the perimeter of one face of the cube is 4a, then the total surface area of the cube is $6\left(\frac{4a}{4}\right)^2 = 6a^2$, which is not $6\left(\frac{a}{4}\right)^2$. Choice D is incorrect because if the perimeter of one face of the cube is 6a, then the total surface area of the cube is $6\left(\frac{6a}{4}\right)^2 = 6\left(\frac{3a}{2}\right)^2$, which is not $6\left(\frac{a}{4}\right)^2$.

Question Difficulty: Hard

Question ID de39858a

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	■■■

ID: de39858a

The function h is defined by $h(x) = a^x + b$, where a and b are positive constants. The graph of $y = h(x)$ in the xy-plane passes through the points $(0, 10)$ and $(-2, \frac{325}{36})$. What is the value of ab?

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. 54

D. 60

ID: de39858a Answer

Correct Answer: C

Rationale

Choice C is correct. It's given that the function h is defined by $h(x) = a^x + b$ and that the graph of $y = h(x)$ in the xy-plane passes through the points $(0, 10)$ and $\left(-2, \frac{325}{36}\right)$. Substituting 0 for x and 10 for $h(x)$ in the equation $h(x) = a^x + b$ yields $10 = a^0 + b$, or $10 = 1 + b$. Subtracting 1 from both sides of this equation yields $9 = b$. Substituting -2 for x and $\frac{325}{36}$ for $h(x)$ in the equation $h(x) = a^x + 9$ yields $\frac{325}{36} = a^{-2} + 9$. Subtracting 9 from both sides of this equation yields $\frac{1}{36} = a^{-2}$, which can be rewritten as $a^2 = 36$. Taking the square root of both sides of this equation yields $a = 6$ and $a = -6$, but because it's given that a is a positive constant, a must equal 6. Because the value of a is 6 and the value of b is 9, the value of ab is $(6)(9)$, or 54.

Choice A is incorrect and may result from finding the value of $a^{-2}b$ rather than the value of ab.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from correctly finding the value of a as 6, but multiplying it by the y-value in the first ordered pair rather than by the value of b.

Question Difficulty: Hard

Question ID 1178f2df

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	■■■

ID: 1178f2df

x	y
21	-8
23	8
25	-8

The table shows three values of x and their corresponding values of y, where $y = f(x) + 4$ and f is a quadratic function. What is the *y*-coordinate of the *y*-intercept of the graph of $y = f(x)$ in the *xy*-plane?

ID: 1178f2df Answer

Correct Answer: -2112

Rationale

The correct answer is $-2{,}112$. It's given that f is a quadratic function. It follows that f can be defined by an equation of the form $f(x) = a(x-h)^2 + k$, where a, h, and k are constants. It's also given that the table shows three values of x and their corresponding values of y, where $y = f(x) + 4$. Substituting $a(x-h)^2 + k$ for $f(x)$ in this equation yields $y = a(x-h)^2 + k + 4$. This equation represents a quadratic relationship between x and y, where $k + 4$ is either the maximum or the minimum value of y, which occurs when $x = h$. For quadratic relationships between x and y, the maximum or minimum value of y occurs at the value of x halfway between any two values of x that have the same corresponding value of y. The table shows that *x*-values of 21 and 25 correspond to the same *y*-value, -8. Since 23 is halfway between 21 and 25, the maximum or minimum value of y occurs at an *x*-value of 23. The table shows that when $x = 23$, $y = 8$. It follows that $h = 23$ and $k + 4 = 8$. Subtracting 4 from both sides of the equation $k + 4 = 8$ yields $k = 4$. Substituting 23 for h and 4 for k in the equation $y = a(x-h)^2 + k + 4$ yields $y = a(x-23)^2 + 4 + 4$, or $y = a(x-23)^2 + 8$. The value of a can be found by substituting any *x*-value and its corresponding *y*-value for x and y, respectively, in this equation. Substituting 25 for x and -8 for y in this equation yields $-8 = a(25-23)^2 + 8$, or $-8 = a(2)^2 + 8$. Subtracting 8 from both sides of this equation yields $-16 = a(2)^2$, or $-16 = 4a$. Dividing both sides of this equation by 4 yields $-4 = a$. Substituting -4 for a, 23 for h, and 4 for k in the equation $f(x) = a(x-h)^2 + k$ yields $f(x) = -4(x-23)^2 + 4$. The *y*-intercept of the graph of $y = f(x)$ in the *xy*-plane is the point on the graph where $x = 0$. Substituting 0 for x in the equation $f(x) = -4(x-23)^2 + 4$ yields $f(0) = -4(0-23)^2 + 4$, or $f(0) = -4(-23)^2 + 4$. This is equivalent to $f(0) = -2{,}112$, so the *y*-intercept of the graph of $y = f(x)$ in the *xy*-plane is $(0, -2{,}112)$. Thus, the *y*-coordinate of the *y*-intercept of the graph of $y = f(x)$ in the *xy*-plane is $-2{,}112$.

Question Difficulty: Hard

Question ID 84e8cc72

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	■■■

ID: 84e8cc72

A quadratic function models the height, in feet, of an object above the ground in terms of the time, in seconds, after the object is launched off an elevated surface. The model indicates the object has an initial height of 10 feet above the ground and reaches its maximum height of $1{,}034$ feet above the ground 8 seconds after being launched. Based on the model, what is the height, in feet, of the object above the ground 10 seconds after being launched?

A. 234

B. 778

C. 970

D. $1{,}014$

ID: 84e8cc72 Answer

Correct Answer: C

Rationale

Choice C is correct. It's given that a quadratic function models the height, in feet, of an object above the ground in terms of the time, in seconds, after the object is launched off an elevated surface. This quadratic function can be defined by an equation of the form $f(x) = a(x-h)^2 + k$, where $f(x)$ is the height of the object x seconds after it was launched, and a, h, and k are constants such that the function reaches its maximum value, k, when $x = h$. Since the model indicates the object reaches its maximum height of $1{,}034$ feet above the ground 8 seconds after being launched, $f(x)$ reaches its maximum value, $1{,}034$, when $x = 8$. Therefore, $k = 1{,}034$ and $h = 8$. Substituting 8 for h and $1{,}034$ for k in the function $f(x) = a(x-h)^2 + k$ yields $f(x) = a(x-8)^2 + 1{,}034$. Since the model indicates the object has an initial height of 10 feet above the ground, the value of $f(x)$ is 10 when $x = 0$. Substituting 0 for x and 10 for $f(x)$ in the equation $f(x) = a(x-8)^2 + 1{,}034$ yields $10 = a(0-8)^2 + 1{,}034$, or $10 = 64a + 1{,}034$. Subtracting $1{,}034$ from both sides of this equation yields $64a = -1{,}024$. Dividing both sides of this equation by 64 yields $a = -16$. Therefore, the model can be represented by the equation $f(x) = -16(x-8)^2 + 1{,}034$. Substituting 10 for x in this equation yields $f(10) = -16(10-8)^2 + 1{,}034$, or $f(10) = 970$. Therefore, based on the model, 10 seconds after being launched, the height of the object above the ground is 970 feet.

Choice A is incorrect and may result from conceptual or calculation errors.

Choice B is incorrect and may result from conceptual or calculation errors.

Choice D is incorrect and may result from conceptual or calculation errors.

Question Difficulty: Hard

Question ID 4b642eef

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	■■■

ID: 4b642eef

The total distance *d*, in meters, traveled by an object moving in a straight line can be modeled by a quadratic function that is defined in terms of *t*, where *t* is the time in seconds. At a time of 10.0 seconds, the total distance traveled by the object is 50.0 meters, and at a time of 20.0 seconds, the total distance traveled by the object is 200.0 meters. If the object was at a distance of 0 meters when $t = 0$, then what is the total distance traveled, in meters, by the object after 30.0 seconds?

ID: 4b642eef Answer

Rationale

The correct answer is 450. The quadratic equation that models this situation can be written in the form $d = at^2 + bt + c$, where a, b, and c are constants. It's given that the distance, d, the object traveled was 0 meters when $t = 0$ seconds. These values can be substituted into the equation to solve for a, b, and c: $0 = a(0)^2 + b(0) + c$. Therefore, $c = 0$, and it follows that $d = at^2 + bt$. Since it's also given that d is 50 when t is 10 and d is 200 when t is 20, these values for d and t can be substituted to create a system of two linear equations: $50 = a(10)^2 + b(10)$ and $200 = a(20)^2 + b(20)$, or $10a + b = 5$ and $20a + b = 10$. Subtracting the first equation from the second equation yields $10a = 5$, or $a = \frac{1}{2}$. Substituting $\frac{1}{2}$ for a in the first equation and solving for b yields $b = 0$. Therefore, the equation that represents this situation is $d = \frac{1}{2}t^2$. Evaluating this function when $t = 30$ seconds yields $d = \frac{1}{2}(30)^2 = 450$, or $d = 450$ meters.

Question Difficulty: Hard

Question ID 9f2ecade

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	■■■

ID: 9f2ecade

$h(x) = x^3 + ax^2 + bx + c$

The function h is defined above, where a, b, and c are integer constants. If the zeros of the function are -5, 6, and 7, what is the value of c ?

ID: 9f2ecade Answer

Rationale

The correct answer is 210. Since -5, 6, and 7 are zeros of the function, the function can be rewritten as $h(x) = (x+5)(x-6)(x-7)$. Expanding the function yields $h(x) = x^3 - 8x^2 - 23x + 210$. Thus, $a = -8$, $b = -23$, and $c = 210$. Therefore, the value of c is 210.

Question Difficulty: Hard

Question ID 6f5540a5

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	■■■

ID: 6f5540a5

Kao measured the temperature of a cup of hot chocolate placed in a room with a constant temperature of 70 degrees Fahrenheit (°F). The temperature of the hot chocolate was 185°F at 6:00 p.m. when it started cooling. The temperature of the hot chocolate was 156°F at 6:05 p.m. and 135°F at 6:10 p.m. The hot chocolate's temperature continued to decrease. Of the following functions, which best models the temperature $T(m)$, in degrees Fahrenheit, of Kao's hot chocolate *m* minutes after it started cooling?

A. $T(m) = 185(1.25)^m$

B. $T(m) = 185(0.85)^m$

C. $T(m) = (185 - 70)(0.75)^{\frac{m}{5}}$

D. $T(m) = 70 + 115(0.75)^{\frac{m}{5}}$

ID: 6f5540a5 Answer

Correct Answer: D

Rationale

Choice D is correct. The hot chocolate cools from 185°F over time, never going lower than the room temperature, 70°F. Since the base of the exponent in this function, 0.75, is less than 1, $T(m)$ decreases as time increases. Using the function, the temperature, in °F, at 6:00 p.m. can be estimated as $T(0)$ and is equal to $70 + 115(0.75)^{\frac{0}{5}} = 185$. The temperature, in °F, at 6:05 p.m. can be estimated as $T(5)$ and is equal to $70 + 115(0.75)^{\frac{5}{5}}$, which is approximately 156°F. Finally, the temperature, in °F, at 6:10 p.m. can be estimated as $T(10)$ and is equal to $70 + 115(0.75)^{\frac{10}{5}}$, which is approximately 135°F. Since these three given values of m and their corresponding values for $T(m)$ can be verified using the function $T(m) = 70 + 115(0.75)^{\frac{m}{5}}$, this is the best function out of the given choices to model the temperature of Kao's hot chocolate after m minutes.

Choice A is incorrect because the base of the exponent, 1.25, results in the value of $T(m)$ increasing over time rather than decreasing. Choice B is incorrect because when m is large enough, $T(m)$ becomes less than 70. Choice C is incorrect because the maximum value of $T(m)$ at 6:00 p.m. is 115°F, not 185°F.

Question Difficulty: Hard

Question ID b73ee6cf

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	■■■

ID: b73ee6cf

The population of a town is currently 50,000, and the population is estimated to increase each year by 3% from the previous year. Which of the following equations can be used to estimate the number of years, t, it will take for the population of the town to reach 60,000 ?

A. $50{,}000 = 60{,}000(0.03)^t$

B. $50{,}000 = 60{,}000(3)^t$

C. $60{,}000 = 50{,}000(0.03)^t$

D. $60{,}000 = 50{,}000(1.03)^t$

ID: b73ee6cf Answer

Correct Answer: D

Rationale

Choice D is correct. Stating that the population will increase each year by 3% from the previous year is equivalent to saying that the population each year will be 103% of the population the year before. Since the initial population is 50,000, the population after t years is given by $50{,}000(1.03)^t$. It follows that the equation $60{,}000 = 50{,}000(1.03)^t$ can be used to estimate the number of years it will take for the population to reach 60,000.

Choice A is incorrect. This equation models how long it will take the population to decrease from 60,000 to 50,000, which is impossible given the growth factor. Choice B is incorrect and may result from misinterpreting a 3% growth as growth by a factor of 3. Additionally, this equation attempts to model how long it will take the population to decrease from 60,000 to 50,000. Choice C is incorrect and may result from misunderstanding how to model percent growth by multiplying the initial amount by a factor greater than 1.

Question Difficulty: Hard

Question ID 7eed640d

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	■■■

ID: 7eed640d

$h(x) = -16x^2 + 100x + 10$

The quadratic function above models the height above the ground h, in feet, of a projectile x seconds after it had been launched vertically. If $y = h(x)$ is graphed in the xy-plane, which of the following represents the real-life meaning of the positive x-intercept of the graph?

A. The initial height of the projectile

B. The maximum height of the projectile

C. The time at which the projectile reaches its maximum height

D. The time at which the projectile hits the ground

ID: 7eed640d Answer

Correct Answer: D

Rationale

Choice D is correct. The positive x-intercept of the graph of $y = h(x)$ is a point (x, y) for which $y = 0$. Since $y = h(x)$ models the height above the ground, in feet, of the projectile, a y-value of 0 must correspond to the height of the projectile when it is 0 feet above ground or, in other words, when the projectile is on the ground. Since x represents the time since the projectile was launched, it follows that the positive x-intercept, $(x, 0)$, represents the time at which the projectile hits the ground.

Choice A is incorrect and may result from misidentifying the y-intercept as a positive x-intercept. Choice B is incorrect and may result from misidentifying the y-value of the vertex of the graph of the function as an x-intercept. Choice C is incorrect and may result from misidentifying the x-value of the vertex of the graph of the function as an x-intercept.

Question Difficulty: Hard

Question ID 43926bd9

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	■■■

ID: 43926bd9

x	$f(x)$
1	a
2	a^5
3	a^9

For the exponential function *f*, the table above shows several values of *x* and their corresponding values of $f(x)$, where *a* is a constant greater than 1. If *k* is a constant and $f(k) = a^{29}$, what is the value of *k* ?

ID: 43926bd9 Answer

Rationale

The correct answer is 8. The values of $f(x)$ for the exponential function f shown in the table increase by a factor of a^4 for each increase of 1 in x. This relationship can be represented by the equation $f(x) = a^{4x+b}$, where b is a constant. It's given that when $x = 2$, $f(x) = a^5$. Substituting 2 for x and a^5 for $f(x)$ into $f(x) = a^{4x+b}$ yields $a^5 = a^{4(2)+b}$. Since $4(2) + b = 5$, it follows that $b = -3$. Thus, an equation that defines the function f is $f(x) = a^{4x-3}$. It follows that the value of k such that $f(k) = a^{29}$ can be found by solving the equation $4k - 3 = 29$, which yields $k = 8$.

Question Difficulty: Hard

Question ID a7711fe8

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	■■■

ID: a7711fe8

What is the minimum value of the function *f* defined by $f(x)=(x-2)^2-4$?

A. -4

B. -2

C. 2

D. 4

ID: a7711fe8 Answer

Correct Answer: A

Rationale

Choice A is correct. The given quadratic function f is in vertex form, $f(x)=(x-h)^2+k$, where (h,k) is the vertex of the graph of $y=f(x)$ in the xy-plane. Therefore, the vertex of the graph of $y=f(x)$ is $(2,-4)$. In addition, the y-coordinate of the vertex represents either the minimum or maximum value of a quadratic function, depending on whether the graph of the function opens upward or downward. Since the leading coefficient of f (the coefficient of the term $(x-2)^2$) is 1, which is positive, the graph of $y=f(x)$ opens upward. It follows that at $x=2$, the minimum value of the function f is -4.

Choice B is incorrect and may result from making a sign error and from using the x-coordinate of the vertex. Choice C is incorrect and may result from using the x-coordinate of the vertex. Choice D is incorrect and may result from making a sign error.

Question Difficulty: Hard

Question ID 1a722d7d

Assessment	Test	Domain	Skill	Difficulty
SAT	Math	Advanced Math	Nonlinear functions	■■■

ID: 1a722d7d

Let the function *p* be defined as $p(x)=\frac{(x-c)^2+160}{2c}$, where *c* is a constant. If $p(c)=10$, what is the value of $p(12)$?

A. 10.00

B. 10.25

C. 10.75

D. 11.00

ID: 1a722d7d Answer

Correct Answer: D

Rationale

Choice D is correct. The value of p(12) depends on the value of the constant c, so the value of c must first be determined. It is given that p(c) = 10. Based on the definition of p, it follows that:

$$p(c)=\frac{(c-c)^2+160}{2c}=10$$

$$\frac{160}{2c}=10$$

$$2c=16$$

$$c=8$$

This means that $p(x)=\frac{(x-8)^2+160}{16}$ for all values of x. Therefore:

$$p(12)=\frac{(12-8)^2+160}{16}$$

$$=\frac{16+160}{16}$$

$$=11$$

Choice A is incorrect. It is the value of $p(8)$, not $p(12)$. Choices B and C are incorrect. If one of these values were correct, then $x = 12$ and the selected value of $p(12)$ could be substituted into the equation to solve for c. However, the values of c that result from choices B and C each result in $p(c) < 10$.

Question Difficulty: Hard